

## 19 Asymptotics for empirical Bayes posteriors

**Catia Scricciolo (Universita Bocconi)**

(Based on joint works with S. Donnet, S. Petrone, V. Rivoirard and J. Rousseau)

Empirical Bayes methods are widely used especially when a data-driven choice of the prior hyper-parameters is adopted as a convenient way to bypass difficulties arising from prior specification. A plug-in estimate solution is expected to lead to inferential answers that are similar, for large sample sizes, to those of fully Bayes hierarchical solutions. Understanding of this commonly believed asymptotic agreement between Bayes and empirical Bayes solutions and, more generally, of the theoretical performance of empirical Bayes methods in non-parametric problems is difficult at this stage, having been so far studied only in a limited number of special cases. In this talk, assuming that (empirical) Bayes methods are evaluated under the assumption that the data are generated from a given “true” parameter, we first present some results on the asymptotic agreement, in terms of merging, between empirical Bayes and Bayesian posterior distributions, which turns out to be equivalent to concentration of the empirical Bayes posterior measure around the “truth”, the main argument used to deal with data-dependent priors being based on the idea of shifting the effect of data in the prior to the likelihood by a suitable parameter transformation. We then refine the analysis providing tools for the study of empirically selected priors in non-parametrics, with a focus on optimal and adaptive posterior concentration rates, stating sufficient conditions that are exemplified in two illustrations: (conditional) density estimation with Dirichlet mixtures, along with the related inverse problem of density deconvolution, and estimation of intensity functions in Aalen models. On the whole, when the hyper-parameter does not affect the posterior concentration rates, there is a lot of flexibility in the choice of the estimator: different choices are indistinguishable in terms of the posterior behavior they induce and the empirical Bayes posterior concentration rates are the same as those of any prior associated with a fixed hyper-parameter. In those cases where the hyper-parameter has an influence on the posterior concentration rates, the choice of the plug-in estimator may instead be crucial and require special care.