



The Impact of Unmodeled Heteroskedasticity on Assessing Measurement Invariance

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Introduction

Assessing **measurement invariance** is an important step in establishing a meaningful comparison of measurements of a latent construct across individuals or groups.

In this study, we examined the **impact of heteroskedasticity** on assessing measurement invariance in single-group models.

We included three single-group models:

- Restricted factor analysis^[1] with product indicators (RFA/PI)
- Restricted factor analysis with latent moderated structural equations (RFA/LMS)
- Moderated nonlinear factor analysis^[2] (MNLFA)

Method

We conducted a **simulation study** to examine the performance of RFA/LMS, RFA/PI, and MNLFA under homoskedasticity and heteroskedasticity.

Manipulated factors:

- Type of noninvariance: scalar or metric
- Total sample size: $N = 100, 200, 500, \text{ or } 1000$
- Type of background variable: categorical or continuous
- Magnitude/direction of common-factor heteroskedasticity
- Magnitude/direction of residual heteroskedasticity

Results

Below is a summary of the **important observations**:

- In fully homoskedastic conditions (i.e., $\beta = 0$ and $\delta = 0$), the Type I error rates of each method were close to the nominal level of .05
- In residual heteroskedastic conditions (i.e., $\beta = 0$ and $\delta \neq 0$), the Type I error rates were close to .05 for all methods
- In common-factor heteroskedastic conditions (i.e., $\beta \neq 0$ and $\delta = 0$), the Type I error rates were severely inflated for RFA/LMS, but well controlled by RFA/PI and MNLFA
- In combined heteroskedastic conditions (i.e., $\beta \neq 0$ and $\delta \neq 0$), the Type I error rates were often severely inflated for RFA/LMS, but generally well controlled by RFA/PI and MNLFA

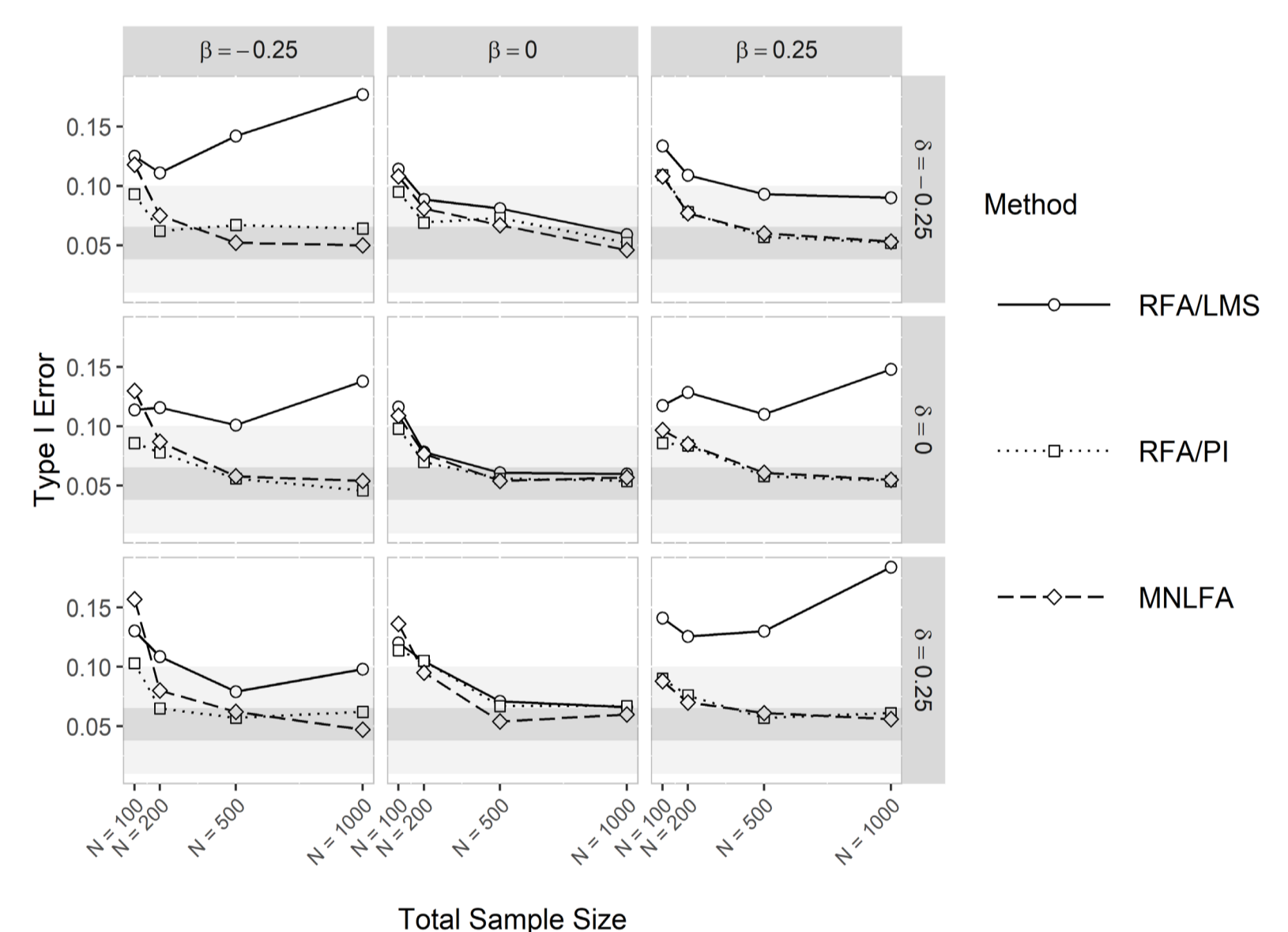


FIGURE 1. Type I error rates in conditions with a categorical background variable

Conclusion

The present study examined the impact of heteroskedasticity on assessing measurement invariance using single-group models.

In the presence of heteroskedastic common factors or residuals, we advise against using the RFA/LMS method because of severely inflated Type I error rates. **RFA/PI and MNLFA are quite robust** to heteroskedasticity because these models (partially) account for it.

Further evaluation of RFA/PI and MNLFA, as well as other recently developed and promising methods^[3-5], for assessing measurement invariance is warranted.

References

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