Black Holes Beyond General Relativity

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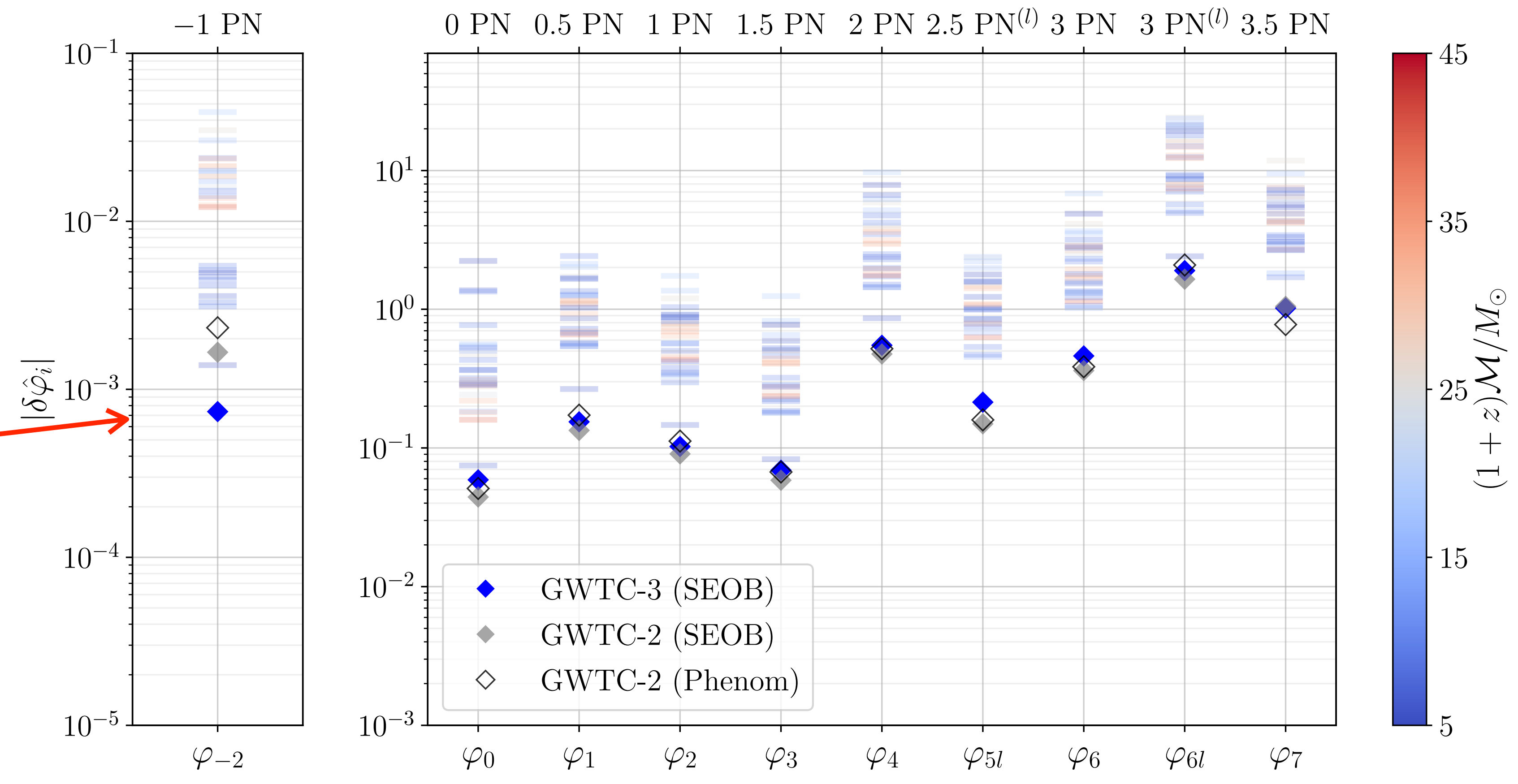
Why should we test GR?

The gravitational interaction *in the weak-field regime* is described by GR with great accuracy (solar system & binary pulsar tests).

However, the observation of the *strong-field* regime of gravity is still at its infancy: GW detectors are only starting to address this question

strongerst bounds are on -1PN corrections,



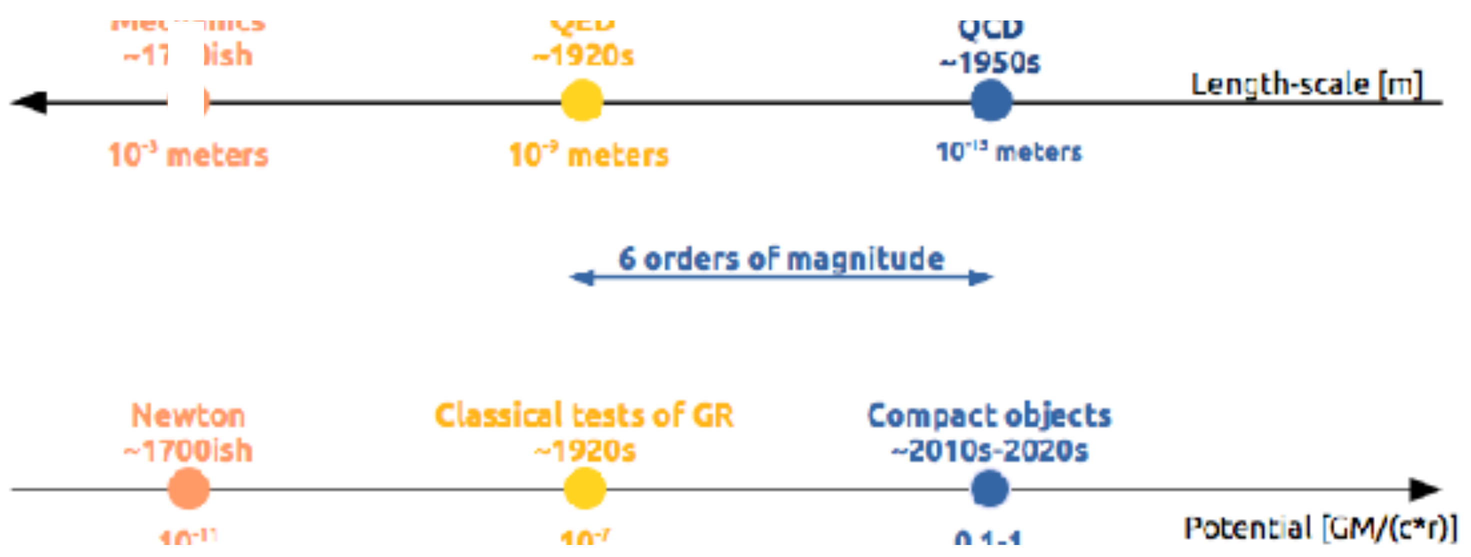
(Abbott et al., PRD '22)

Is it worth testing GR in the strong-field regime?

- There is no fundamental reason to believe that gravity behaves in the same way as the weak-field regime

- Theoretical issues (consistency with the quantum world, singularities, BH information paradox)

- Observational issues (still misterious nature of dark matter & dark energy, Hubble tension)



Credits: Pani, '15

BHs are the perfect probes to understand the nature of the gravitational interaction!

- They are the 'hydrogen atom' of gravity, the simplest gravitating system in the Universe!

  - No-hair theorems: *stationary* BHs in GR (and in several extensions thereof) are described by the Kerr solution.
    They only depend on their mass and angular momentum (+ electric charge, non relevant in astrophysical BHs).

  - Dynamical BHs are not so simple, but still PN expansions & perturbation theory show that Kerr is still a good approximation.
    (e.g. Cardoso & Gualtieri '16)
    Matter (ordinary and possibly dark) and non-gravitational fields lead at most to marginal modification on BH processes, which still can be important when very high precision is required (the so-called *environmental effects).*

  - Neutron stars are completely different: the GW and EM emission heavily depend on non-gravitational effects.
    This can be good for multimessenger astronomy (GW170871 vs. GW150914: coalescing BBHs are very *clean* systems);
    not so good for looking for tiny deviations from GR: an observed *new physics* effect may be concealed by our ignorance on the NS equation of state.

- BHs can produce the strongest gravitational fields after the Big Bang!

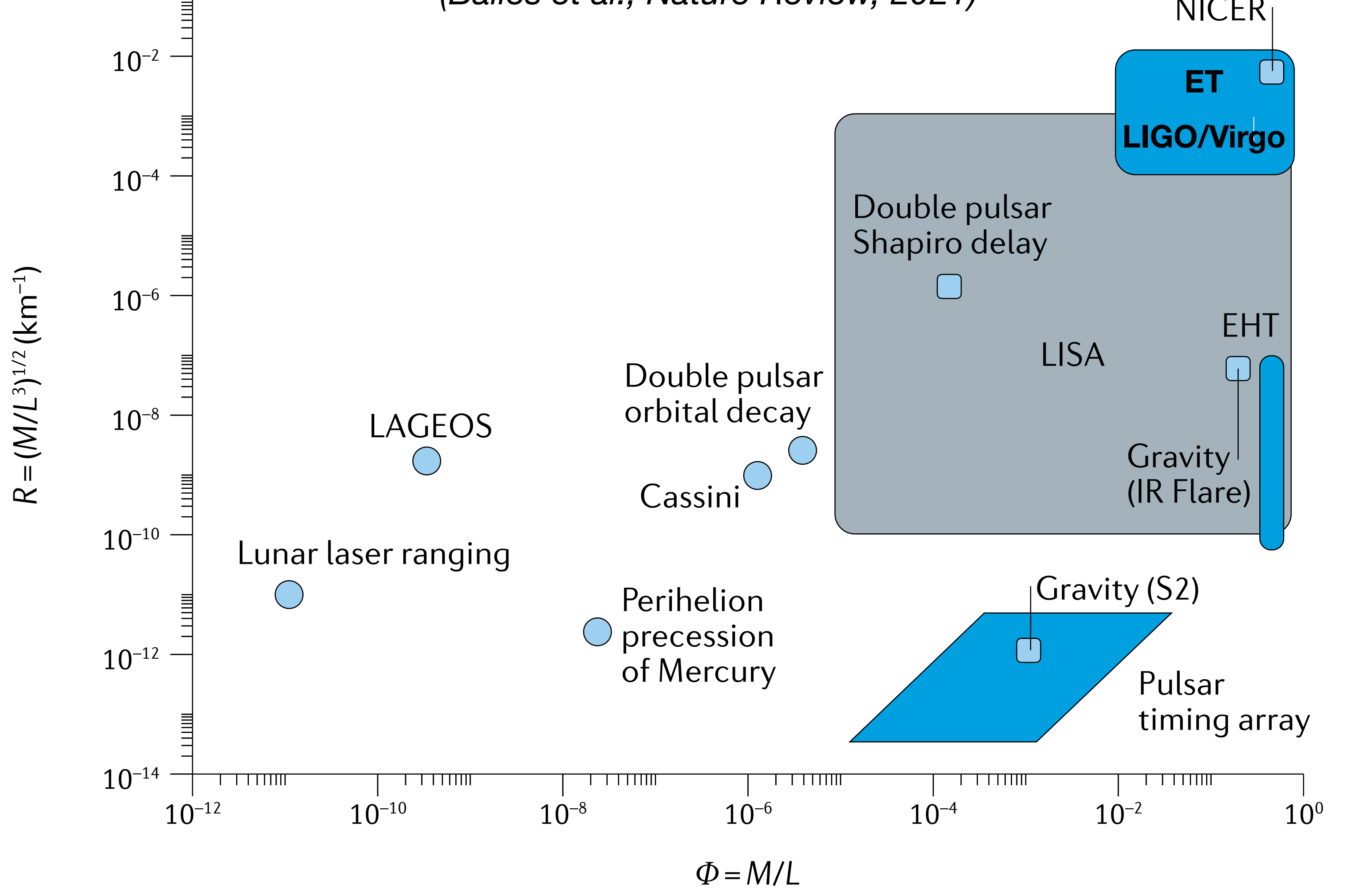
observations of BH processes by ET/CE, LISA, EHT can probe
the larger values of the *gravitational potential* and/or of the *gravitational curvature*

Complementary tests of different phenomenologies:

- Dynamics of gravitational theory: binary BH (BBH) coalescence
  (GW detectors: LVK, ET/CE, LISA, PTA)
- Stationary BH spacetime: BH shadow (EHT) (see talk by De Laurentis yesterday)

I will mainly (but not exclusively) focus on tests on BBH coalescence.

(Bailes et al., Nature Review, 2021)



Testing gravity with BHs

- Bottom-up approaches:

Choose phenomenology to be studied and the quantities more appropriate to study it; devise a parametrization of these quantities, and use observations to set bounds to the parameters.

Example: PPE (parametrized post-Einstenian expansion *Yunes & Pretorius 2009*)
parametrizes gravitational waveform from BBH coalescence

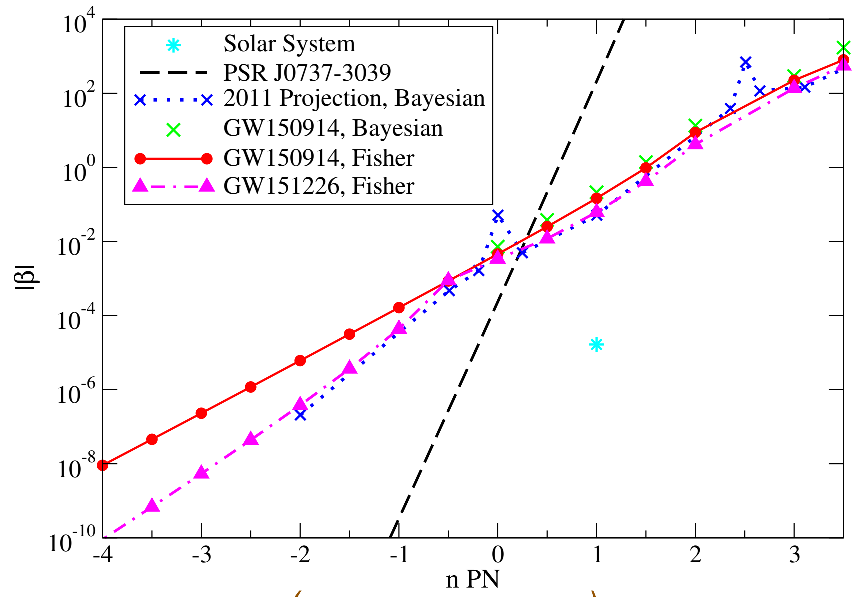
$$h(f) = A_{GR}(f)(1 + \alpha x^a)e^{i\Psi_{GR}(f) + i\beta x^b}$$

ppE parameters:

α,β (=0 inGR): amplitude of modification;

a,b: PN order

mapping: (α,β,a,b) <=> specific theories



(Yunes et al., PRD '16)

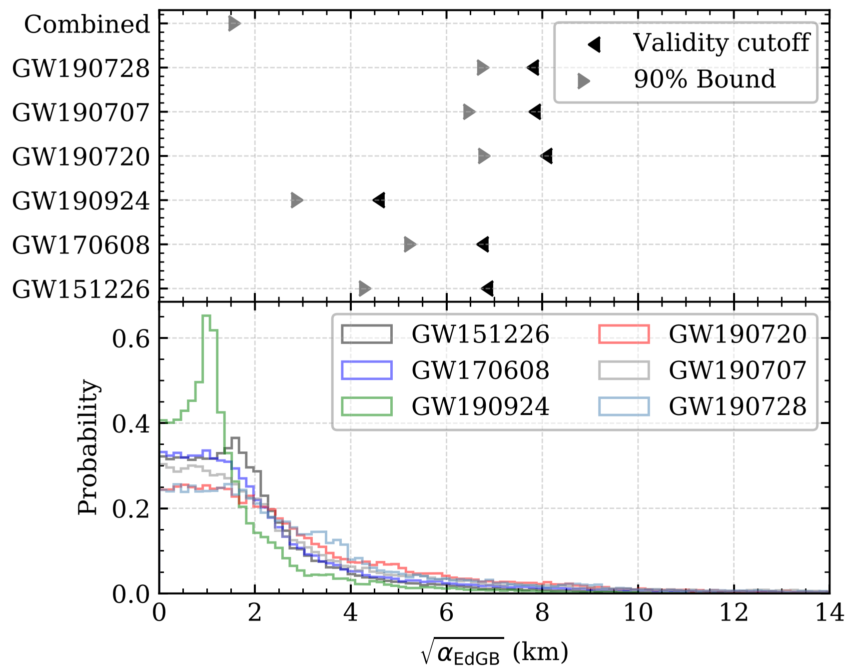
Other examples: parametrization of the stationary BH spacetime (e.g. *Johannsen & Psaltis 2011, Konoplya et al. 2016*)

Caution! Such parametrizations are useful for analyzing BH shadows, not for BBH coalescence or BH oscillations, which depend on the dynamics of the theory!

- Top-down approaches:

Look at possible "test-bed" modified gravity theories, possibly inspired by fundamental physics considerations; work out observational consequences of such modifications (which typically depend on *coupling parameters)* and compare with observations, setting bounds on coupling parameters (or measure them!)

Example: EdGB gravity (Einstein-dilaton Gauss-Bonnet *Kanti et al. '96*)



(Perkins et al., PRD '21)

Both approaches have limitations:

- bottom-up parametrization may miss unexpected features. It is difficult to go beyond null tests without an insight of the possible deviations we may expect;
- top-down analyses depend on arbitrary choices of specific theories.

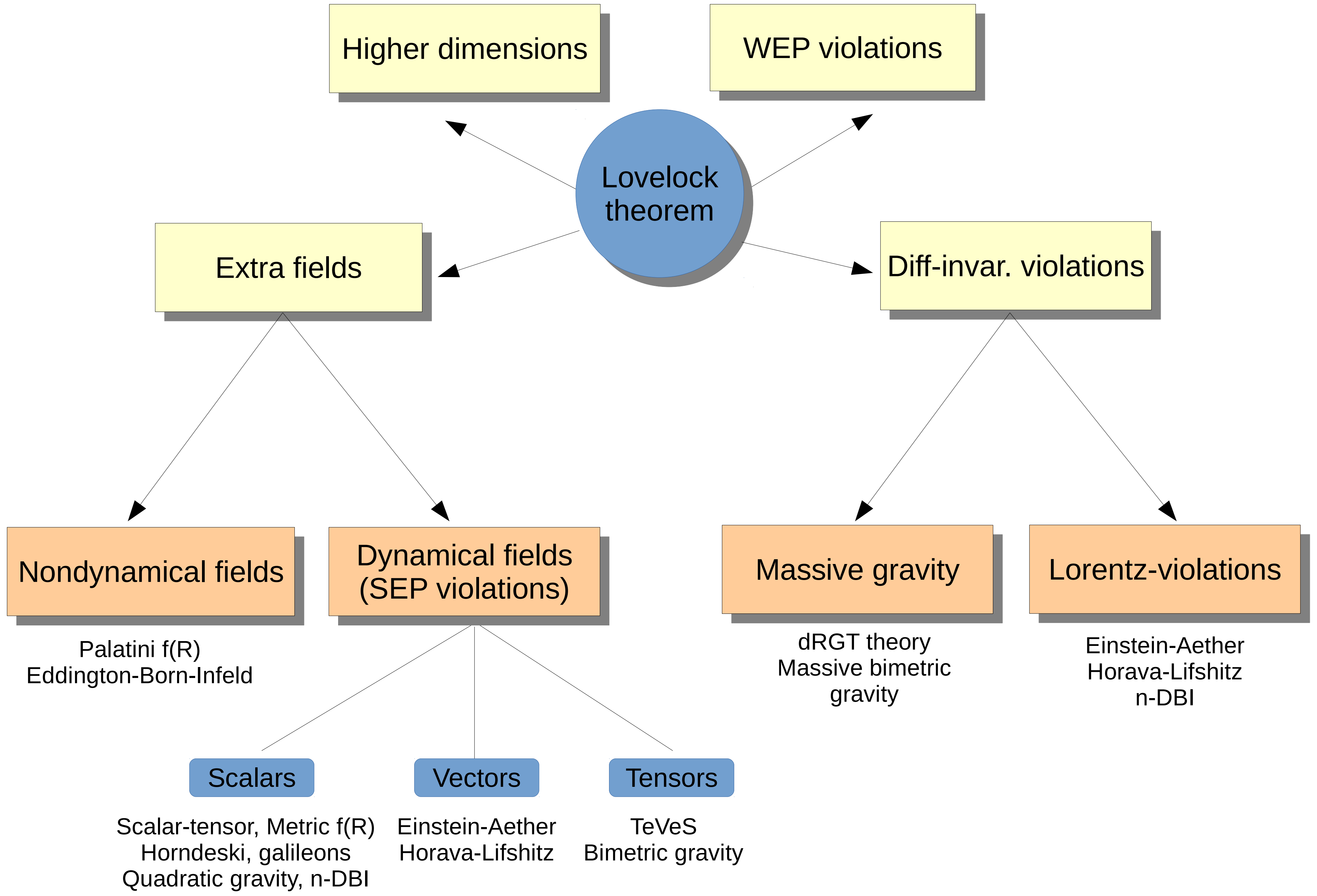
The best strategy may be a combination of the two: we need to study at least examples of top-down waveforms to understand how classes of GR deviations may affect the signal. And use this information to perform consistent tests, and to devise bottom-up parametrizations.

Problem: we don't know the full BBH waveform for any gravity theory beyond GR! So, when we parametize deviations, we move in the dark…

Modified gravity theories

Several modifications of GR have been proposed so far.
A guiding princniple to navigate among them can be the following:
if we find experimental hints of GR modifications,
which of the assumptions underlying GR should be abandoned?

Lovelock's theorem states that GR is the unique theory of gravity under certain conditions (four dimensions, equivalence principle, diffeomorphism invariance, no fields besides the metric). Violations to one or another of them give rise to the different possible modified theories of gravity.



Berti et al., CQG 2015

Main theories under study:

- Theories with extra fields in the gravitational sector; in particular the simplest case of an additional scalar field (*scalar-tensor gravity*)

  This is the most extensively studied class of modified gravity theories, because
  - it is simple;
  - fundamental motivations (e.g. string theory);
  - several other theories can be reformulated in terms of additional fields
  - some of them may provide alternative intepretation of cosmology (DM, DE)

some theories belong to both classes: both
- scalar field in the gravitational sector
- polinomials in curvature

- Higher-derivative gravity theories, arising from EFT expansion: action including polinomials in curvature tensor
- f(R) theories (may provide alternative interpretation of DE) equivalent to scalar-tensor theories!
- Theories with violation of Lorentz invariance (Einstein-Aether theory, Horava gravity, etc…)
- Theories with large extra dimensions (we live in a four-dimensional subspace of a higher-dimensional space)
- Massive graviton theories
- etc…

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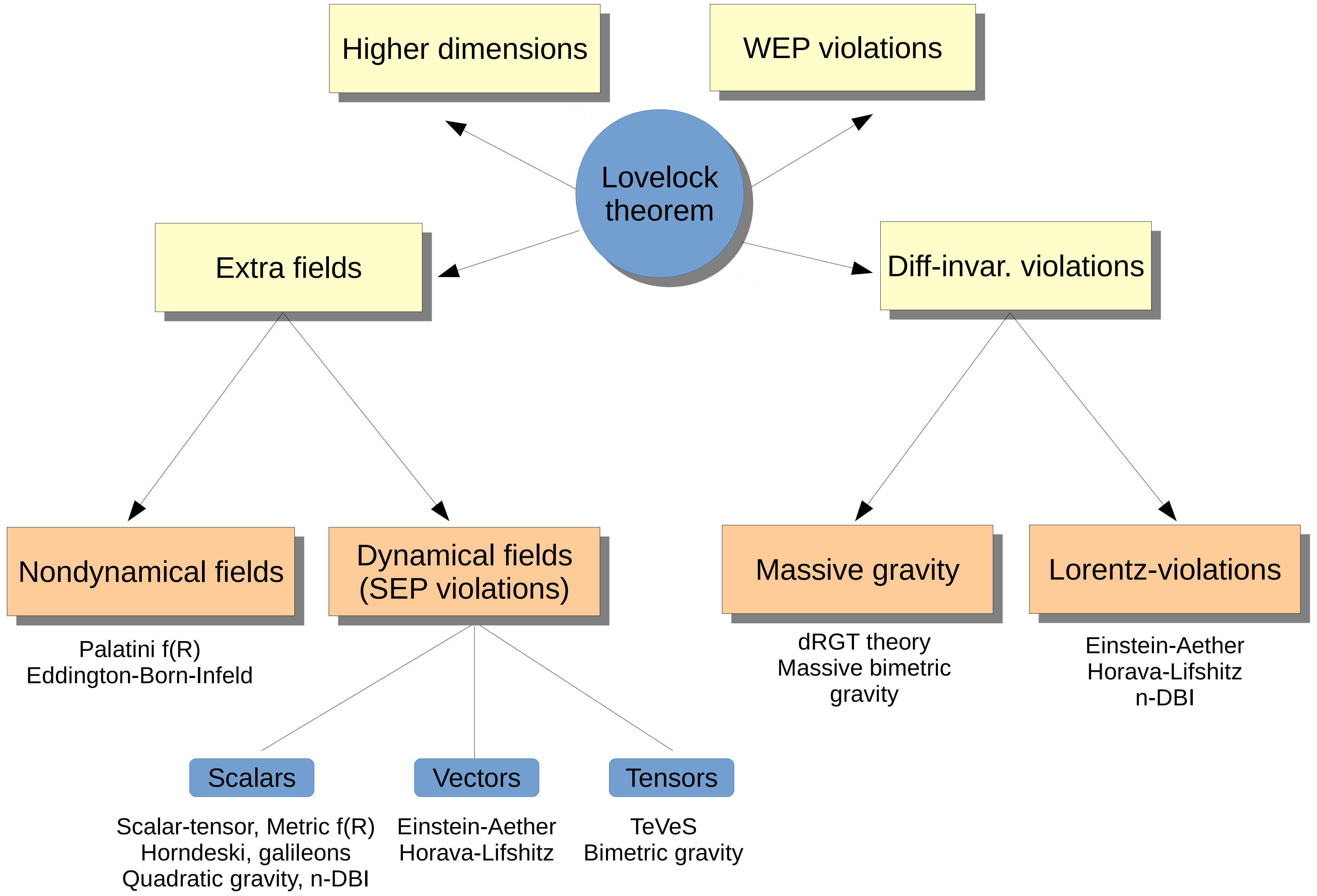

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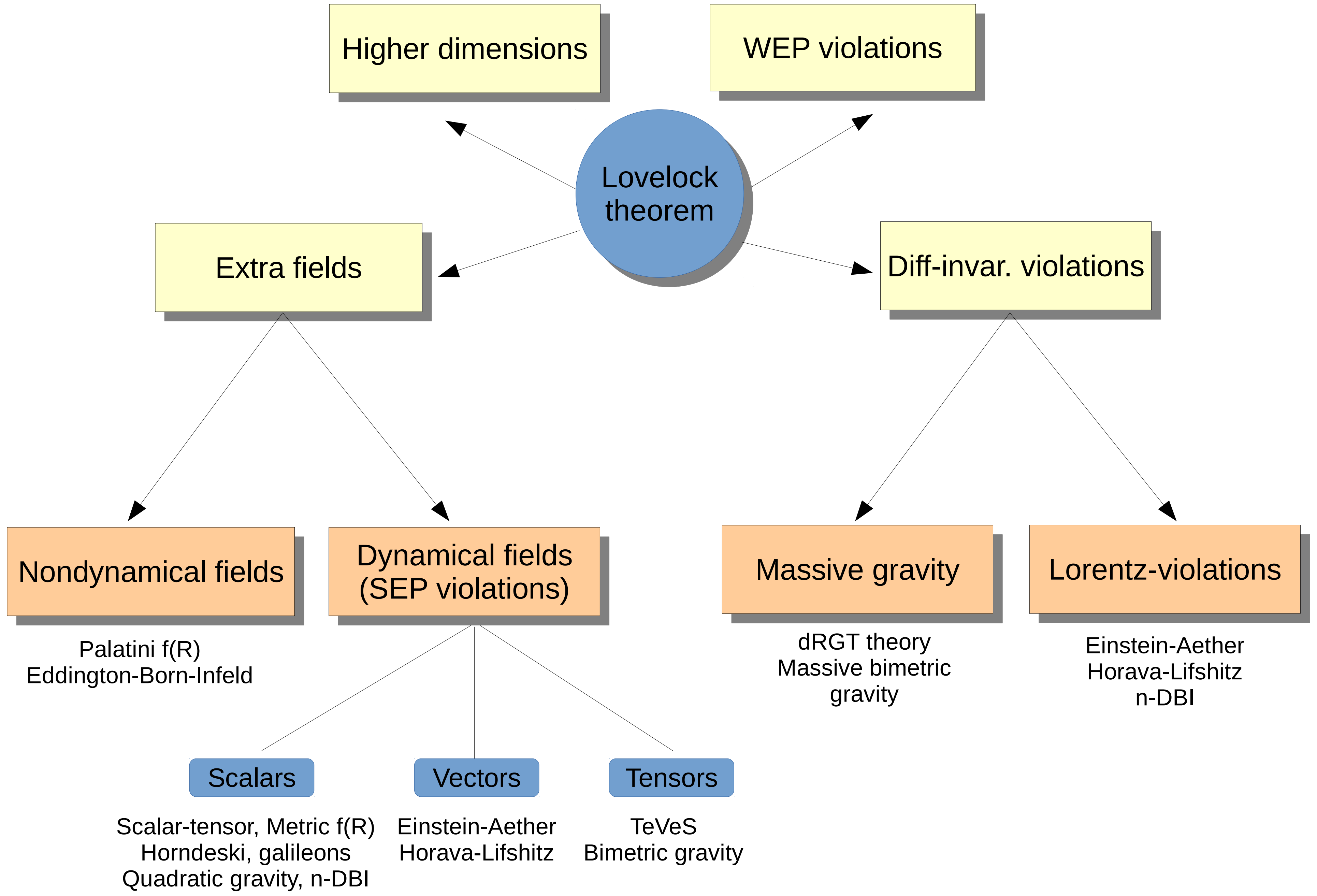

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Scalar-tensor gravity

- Simplest ("Bergmann-Wagoner") scalar-tensor theories:
  (e.g. Fuji & Maeda book 2003)

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\phi R-\frac{\omega(\phi)}{\phi}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right)-U(\phi)\right]+S_M[\Psi,g_{\mu\nu}]$$

non-miminal coupling between scalar field and gravity

- The simplest case is Brans-Dicke gravity, proposed ~60 years ago
- f(R) gravity can be reformulated as a scalar-tensor theory in this class
- Problem for phenomenology: no-hair theorems apply to these theories, i.e. stationary BHs are Kerr!
- For vacuum spacetimes (ψ=0) this theory is equivalent by conformal rescaling to GR + minimally coupled scalar field ("Einstein frame")

Does this mean that BHs are useless to probe such theories?

Not necessarily! if scalar field is *ultralight massive* (m~10^{-13}-10^{-22} eV), BH may have a scalar field cloud which, although non-stationary, is metastable and long-lived! (possible DM interpretation)
(e.g. Barranco et al. 2012, Cardoso et al. 2022, Figueras & França, 2022)

- Quadratic gravity: scalar-tensor theories with quadratic curvature terms:

$$S=\frac{1}{16\pi G}\int\sqrt{-g}d^4x\Big[R-2\nabla_a\phi\nabla^a\phi-V(\phi)+f_1(\phi)R^2$$
$$+f_2(\phi)R_{\mu\nu}R^{\mu\nu}+f_3(\phi)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+f_4(\phi)^*RR\Big]$$
$$+S_{\rm mat}\left[\Psi,\,\gamma(\phi)g_{\mu\nu}\right]\,,$$

E-H action is the first term in an expansion containing all possible curvature terms, as suggested by low-energy effective field theories.

Typically, quadratic gravity theories have ghosts and other pathologies, with the exception of a particular combination: scalar Gauss-Bonnet (sGB) gravity

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\alpha_{GB}f(\phi)\mathcal{R}^2_{GB}\right\}\qquad\left(\mathcal{R}^2_{GB}=R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2\right)$$

(Kanti et al. 1995)

Besides that, the other theory leading to new BH solutions is dynamical Chern-Simons (dCS) gravity, which can be considered in an EFT framework (small coupling)

(Alexander & Yunes 2009)

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\alpha_{CS}\phi\,{}^*RR\right\}\qquad\left({}^*RR=\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\mu\nu}R^{\mu\nu}{}_{\gamma\delta}\right)$$

Scalar-tensor gravity

- Horndeski gravity: the most general scalar-tensor theoriy with second-order-in-time field equations (and thus free from the so-called Ostrogradski instability):

$$S = \int d^4x\sqrt{-g}\,\{K(\phi,X) - G_3(\phi,x)\Box\phi + G_4(\phi,X)R + G_{4,X}\left((\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right)$$

$$+G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - G_{5,X}\frac{1}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\rho\phi)(\nabla^\mu\nabla_\sigma\phi)(\nabla^\rho\nabla^\sigma\phi)\right]\Big\}$$

$$+ S_{matter}(\psi_m, g_{\mu\nu}) \qquad (X \equiv \partial_\mu\phi\partial^\mu\phi)$$

- K-essence : $K \neq 0$ $G_3 = G_5 = 0$ $G_4 = G_4(\phi)$
  - if $K = X + f(\phi)X^2$ it is called *quadratic gravity*
  - includes, among the others, Bergmann-Wagoner theories
  - includes cosmological models of DE
  - no-hair theorems apply: stationary BHs are Kerr!

- cubic gravity: $K \neq 0$ $G_3 \neq 0$ $G_4 = G_5 = 0$
  - include cosmological models of DE
  - no-hair theorems apply: stationary BHs are Kerr!

- sGB gravity: $K = X/2$ $G_3 = 0$ $G_4 = 1/2$ $G_5 = G_5(\phi,X)$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB}f(\phi)\mathcal{R}^2_{GB}\right\} \qquad (\mathcal{R}^2_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

😀 no-hair theorems do not apply: stationary BH can have a non-trivial scalar field profile!

😟 the scalar field can not describe DE: that would lead to a speed of GW $\neq$ c, excluded by the observation of GW170817 with enormous accuracy

Scalar-tensor gravity

Few more words about sGB gravity

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{GB} f(\phi)\mathcal{R}^2_{GB}\right\} \qquad (\mathcal{R}^2_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2)$$

- they are the simplest scalar-tensor gravity theories in which no-hair theorems do not apply
- GR deviations appear at large curvatures => no constraints from binary pulsars, need GW
- in some of these theories, spontaneous scalarization occurs: a sort of phase transition in which a BH grows a non-trivial scalar field configuration
- these theories can arise from string-theory compactifications ($f(\phi)=e^\phi$: Einstein-dilaton Gauss-Bonnet)
- the Gauss-Bonnet term can be seen as an effective-field-theory contribution

but:

- in sGB gravity, scalar field can not have cosmological interpretation (ruled out by GW170817)
- dimensionful coupling constant α_{GB} needs to be at least of order km^2 for observable GW signature (EFT interpretation requires new scale in the theory besides l_P)

Some of these properties are shared by dCS gravity:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha_{CS}\phi\, {}^*RR\right\} \qquad ({}^*RR = \epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\mu\nu}R^{\mu\nu}{}_{\gamma\delta})$$

- no-hair theorems do not apply
- can be seen as an EFT contribution
- naturally arise from string-theory compactifications
- no cosmological interpretation as DE

differently from sGB:

- leads to *parity violation* in gravity
- does not belong to Horndesky class: it is ill-behaved if considered as a full theory (only EFT interpretation allowed)!

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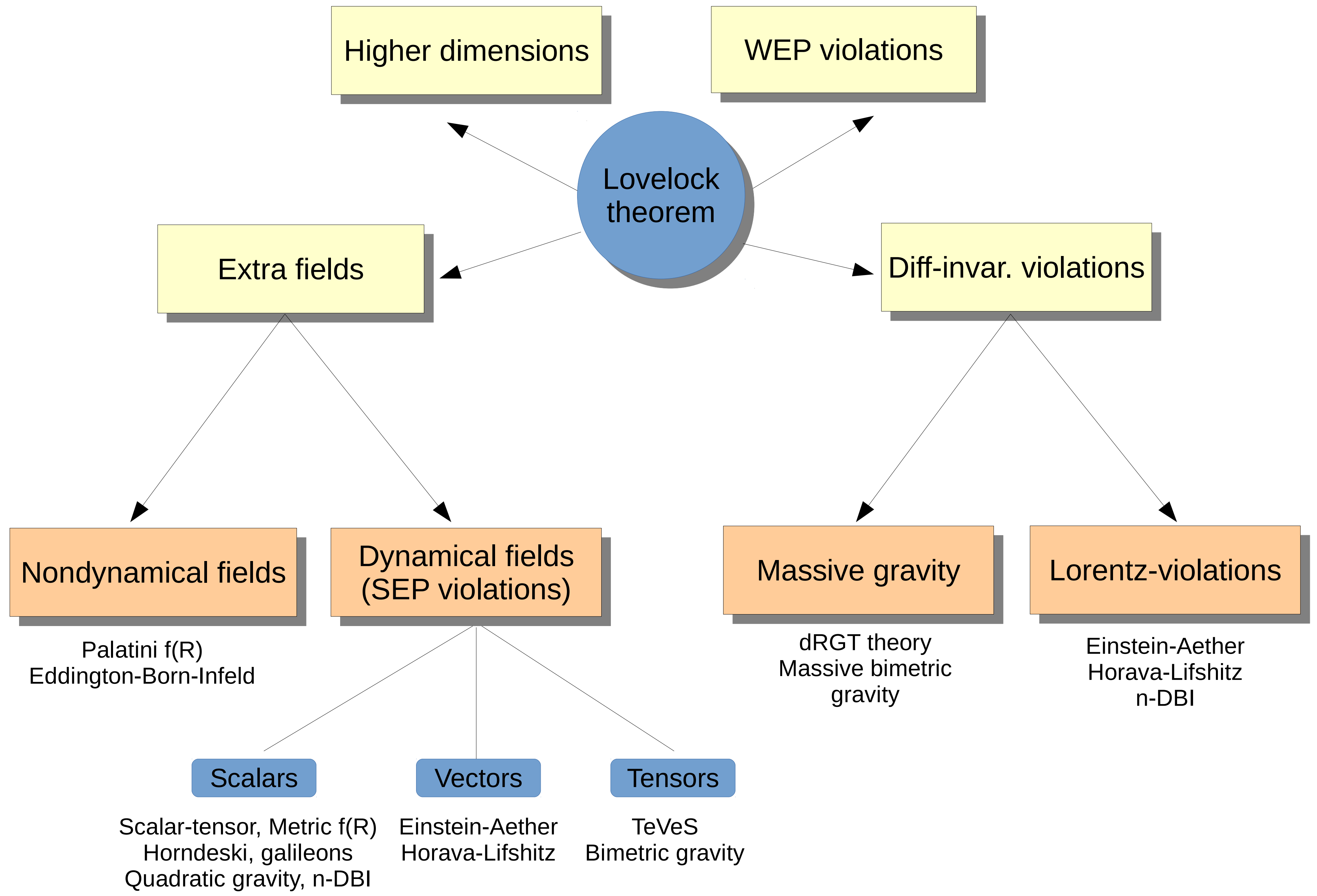

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- **Higher-derivative** gravity theories, arising from EFT expansion: action including polinomials in curvature tensor
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Higher-derivative gravity theories: EFT

(Cardoso et al. 2018, Cano & Rupierez 2020)

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R + \ell^4[\lambda_{\rm even}\mathcal{R}^3 + \lambda_{\rm odd}\tilde{\mathcal{R}}^3] + \ell^6[\epsilon_1\mathcal{C}^2 + \epsilon_2\tilde{\mathcal{C}}^2 + \epsilon_3\tilde{\mathcal{C}}\mathcal{C}]\right)$$

$$\mathcal{R}^3 = R^{\rho\sigma}{}_{\mu\nu}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} \quad , \quad \tilde{\mathcal{R}}^3 = R^{\rho\sigma}{}_{\mu\nu}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu} \, , \quad \mathcal{C} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \quad , \quad \tilde{\mathcal{C}} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} \, ,$$

- This class of theories naturally arise in an EFT framework, assuming that GR is the lowest order in a polynomial expansion of the curvature tensor.
- Since there is no scalar field, the terms quadratic in the dimensionful coupling (like in sGB and dCS) are trivial and thus the expansion starts at cubic order in ℓ
- No obvious cosmological interpretation
- In these theories the no-hair theorem is not satisifed: BHs are different from those of Kerr!
- Same problem as in sGB and dCS: deviations observable by GW detectors only if ℓ is a new fundamental scale ($\ell >> \ell_{\rm Pl}$)

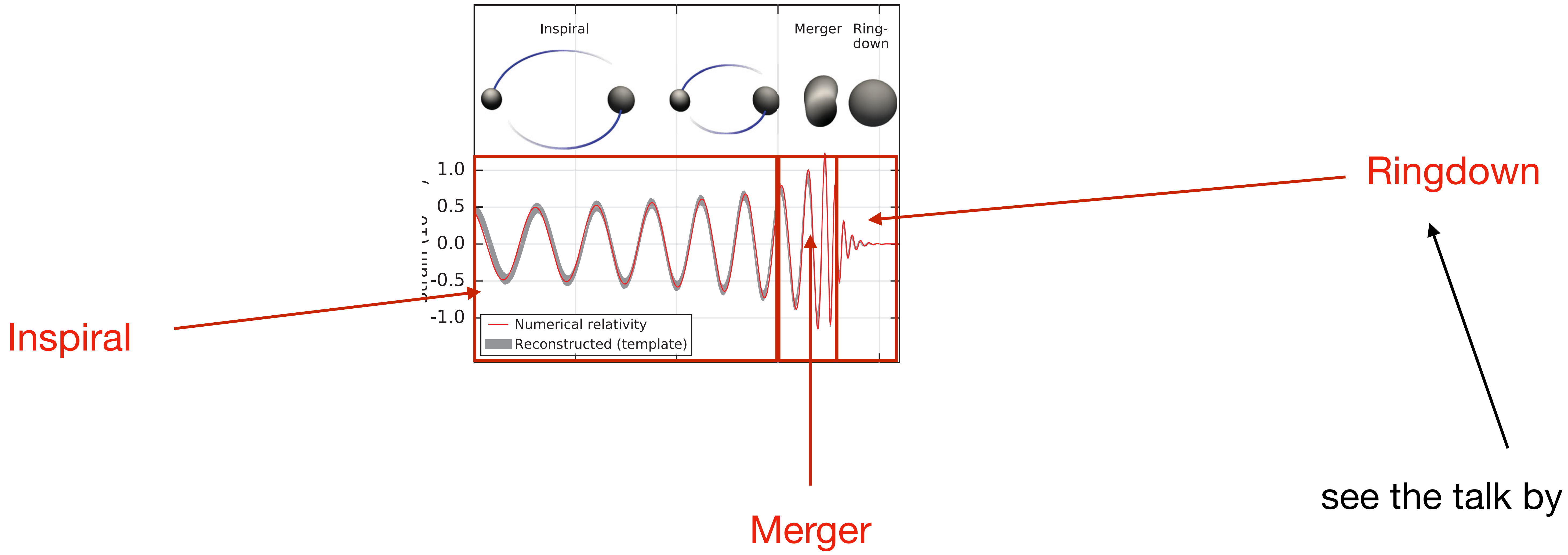
What to we know of BHs in these theories?

This question can be addressed to:

- Stationary BHs

  Relevant not only to observation of stationary BHs themselves (e.g. BH shadows) but also as a first step to understand BBH coalescences!

- BBH coalescences:



see the talk by Doneva!

Stationary BHs beyond GR

- theories with no-hair theorem (Bergmann-Wagoner scalar-tensor, K-essence, cubic Horndesky, f(R)…): Kerr solution!

- sGB gravity $S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\alpha_{GB}f(\phi)\mathcal{R}^2_{GB}\right\}$ $\left(\mathcal{R}^2_{GB}=R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2\right)$ solution depends on f(ϕ):

- $f(\phi) = e^{\phi}$: Einstein-dilaton Gauss-Bonnet gravity
(*Kanti et al. 1995, Pani & Cardoso 2009, Kleihaus et al. 2011*)

- $f(\phi) = \phi$: shift-symmetric Gauss-Bonnet gravity
(*Sotiriou & Zhou 2014*)

- BH solutions have GR corrections of the order of $\zeta=\frac{\alpha_{\rm GB}}{M^2}$
(radius of ISCO, horizon, etc. have O(ζ) corrections)

- solutions exist for $0<\zeta<0.619$ (small modifications for spinning BHs)
thus, for a given coupling, there is a minimum mass!
Smaller masses lead to *naked singularities!*

Since BHs do exist for $M\sim5M_{\rm sun}$, we have a theoretical bound on $\alpha_{\rm GB}$.
(current LVK bounds are of the same order: $\sqrt{\alpha_{GB}^{max}}\sim1\,{\rm km}$)
More massive BHs have tiny corrections!
This makes SMBHs irrelevant, and even most LVK BHs not so relevant.
This because these are large curvature deviations, and massive BHs have small curvature!

- $f(\phi) = \phi^2$, $f(\phi) = (1-e^{-\phi^2})$: theories with BH spontaneous scalarization
(*Doneva et al. 2018, Silva et al. 2018, Dima & Yazadjiev 2020, Herdeiro et al. 2021*)

In these theories, *both* Kerr and scalarized BHs can be solutions.
In certain ranges of mass or of spin,
Kerr BHs spontaneously develop a non-trivial scalar field profile.
There is no minimum mass for the *existence* of these solutions,
but there are indications that there is a minimum mass for their *stability*
so the same problems arise!

Stationary BHs beyond GR

- dCS gravity $$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+\alpha_{CS}\phi\,{}^*RR\right\}$$ $$({}^*RR=\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\mu\nu}R^{\mu\nu}{}_{\gamma\delta})$$

(Alexander & Yunes 2009)

- *rotating* BH solutions have GR corrections of the order of $\zeta=\frac{\alpha_{CS}}{M^2}$

  (radius of ISCO, horizon, etc. have O(ζ) corrections)

- the theory is only meaningful for ζ<<1
- current bounds of the same order as sGB: $\sqrt{\alpha_{CS}^{max}}\sim 1\,\mathrm{km}$

- higher-derivative EFT gravity $$\mathcal{S}=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R+\ell^4[\lambda_{\rm even}\mathcal{R}^3+\lambda_{\rm odd}\tilde{\mathcal{R}}^3]+\ell^6[\epsilon_1\mathcal{C}^2+\epsilon_2\tilde{\mathcal{C}}^2+\epsilon_3\tilde{\mathcal{C}}\mathcal{C}]\right)$$

(Cardoso et al. 2018, Cano & Rupierez 2020)

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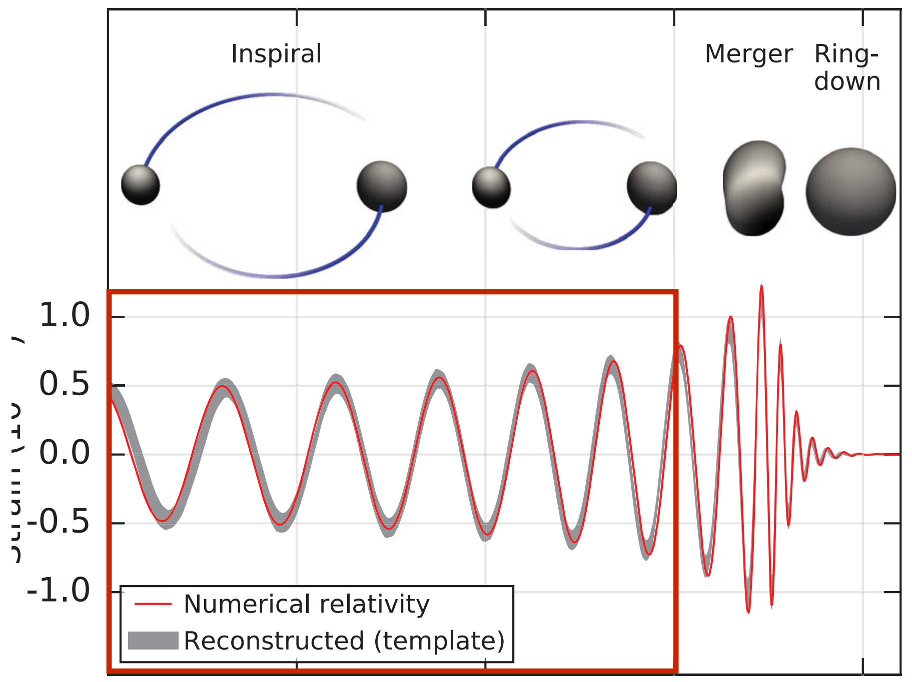
- BH solutions have GR corrections of the order of $\left(\frac{\ell}{M}\right)^4$
- no evidence of a minimum mass; however, since we do not expect $\frac{\ell}{M}>1$ to be consistent with the EFT expansion, even fror ~5M_{sun} BHs, again we expect these deviations to be highly suppressed for most LVK BHs, and enormously soppressed for SMBHs.

This because these are large curvature deviations, and massive BHs have small curvature!

BBH inspiral beyond GR



- in GR, the early inspiral of a BBH is described through post-Newtonian (PN) expansions

- even in GR, PN expansions are not accurate enough for late inspiral, where v~c and the perturbative expansions is ill-defined: we need a phenomenological/EoB waveform, with calibration from numerical relavity (NR) simulations

- beyond GR, the inspiral of a BBH is potentially a very important probe: even in the gravitational field is weak, a scalar field (if present) would lead to dipolar emission, which does not occur all in GR (a "smoking gun"): GW emission in GR starts with quadrupole; for this reason, dipolar emission is magnified in a PN expansion (-1PN order). Unfortunately, in several theories BHs do not have scalar fields…
  - rougly speaking, dipolar emission speeds up the inspiral; thus, even without detecting the scalar field, we can see an effect (at a different PN order than others)

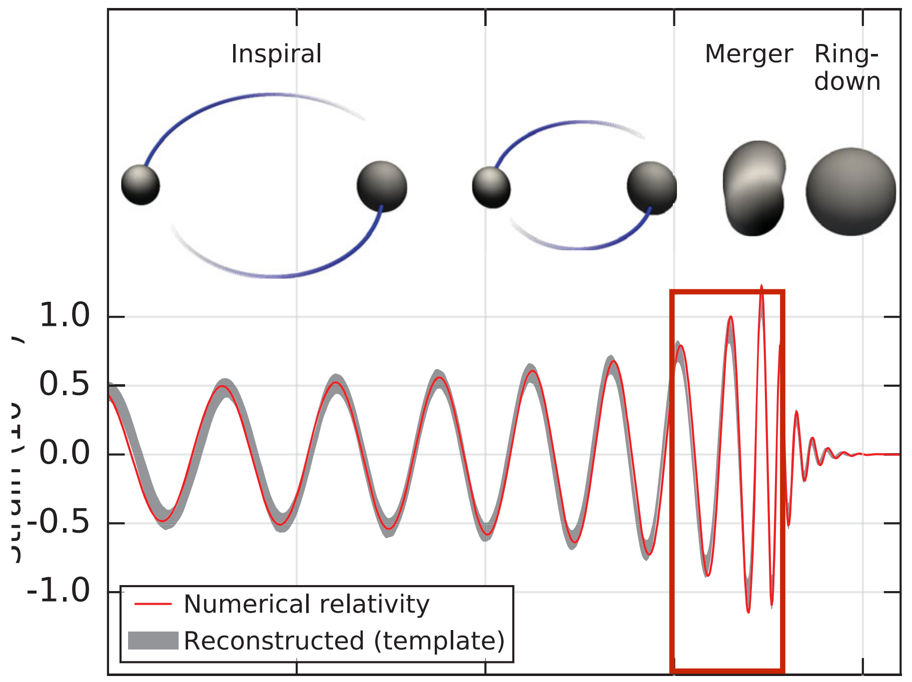
- PN expansions have been generalized beyond GR in some theories: Bergmann-Wagoner theories, sGB gravity, dCS gravity;
  (Yagi et al., 2012; Lang, 2014; Julie & Berti 2019; Shiralilou et al. 2022; Bernard et al. 2022)

  these computations allowed to use LVK data to get the best constraints to date to sGB gravity and dCS gravity
  (Perkins et al. 2021)

- we still do not have any phenomenological/EoB waveform beyond GR!
  For this, we need to better understand the merger

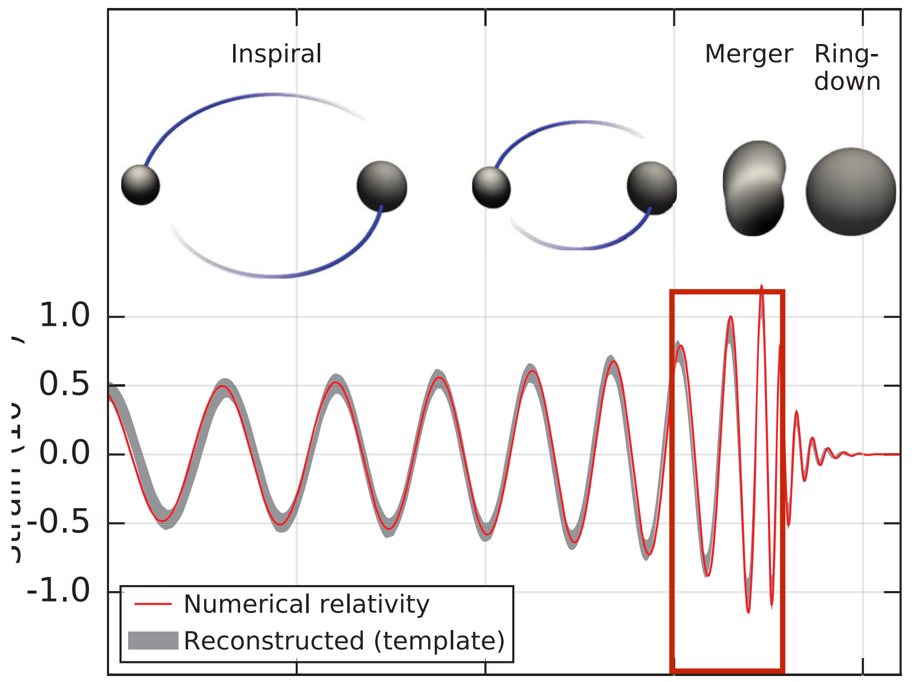
- moreover, we only know PN expansions in scalar-tensor gravity theories

BBH merger beyond GR



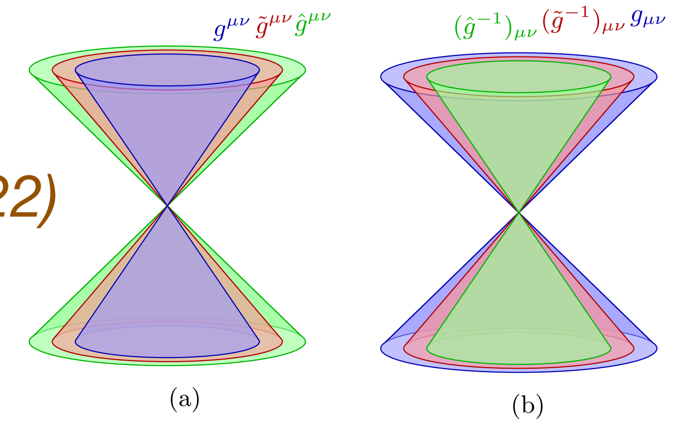
- In GR, the development of NR simulation took almost half a century! Extending these simulations to modified gravity theories is a formidable task!
- The first problem one encounters is the requisite of well-posedness: *a well-posed problem admits a unique solution with continuous dependence on initial data;* this requires *strong hyperbolicity of field equations*
- Even if the theory is perfectly sound, it is not obvious to find a well-posed formulation (even in GR, the ADM formulation is not well-posed!)
- On the other hand, if the theory is ill-defined (like dCS gravity, which only makes sense within an EFT expansion), for sure no well-posed formulation is possible!

BBH merger beyond GR



Beyond GR, well posedness has been proven in:

- Bergmann-Wagoner ST theories (because they can be reformulated as GR with a scalar field coupled with "matter") [note: BHs are like in GR!]
- Quadratic and cubic Horndeski gravity *(Kovacs, 2019)* [note: BHs are like in GR!]
- General Horndesky gravity (including sGB) at *weak coupling* *(Kovacs & Reall 2020; Aresté Saló et al. 2022)*



Remark: if a theory is expanded in the coupling constant and solved order by order, it is well-posed at each given order!

This approach is meaningful in an EFT framework: higher-order terms may remain small, at least for some evolutions and for some time.

$$\Phi = \sum_{k=0}^{\infty} \frac{1}{k!} \epsilon^k \Phi^{(k)} ,$$

$$g_{ab} = g_{ab}^{(0)} + \sum_{k=1}^{\infty} \frac{1}{k!} \epsilon^k h_{ab}^{(k)} .$$

$$\epsilon^0 : \quad G_{ab}^{(0)} = \frac{1}{2} T_{ab}^{(0)} , \quad \Box^{(0)} \Phi^{(0)} = 0 ,$$

$$\epsilon^1 : \quad G_{ab}^{(1)} = \frac{1}{2} T_{ab}^{(1)} - 4M^2 \mathcal{G}_{ab}^{(0)} ,$$

$$\Box^{(0)} \Phi^{(1)} = -\Box^{(1)} \Phi^{(0)} - 8M^2 f'_{(0)} \mathcal{R}_{\rm GB}^{(0)} ,$$

Witek et al., 2019, Okounkova et al. 2017, 2019, 2020

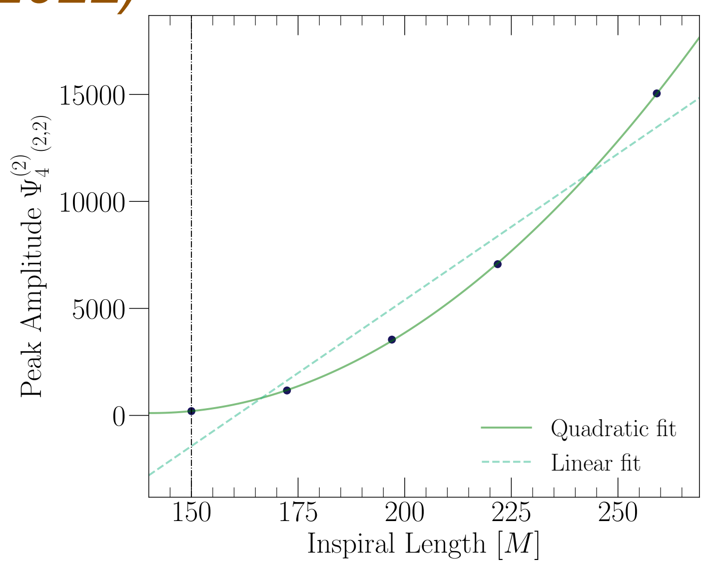
BBH merger beyond GR: NR simulations

- Order by order in a perturbative expansion in the coupling, sGB gravity and dCS gravity

  (Witek et al., 2019, Okounkova et al. 2017, 2019, 2020, 2022)

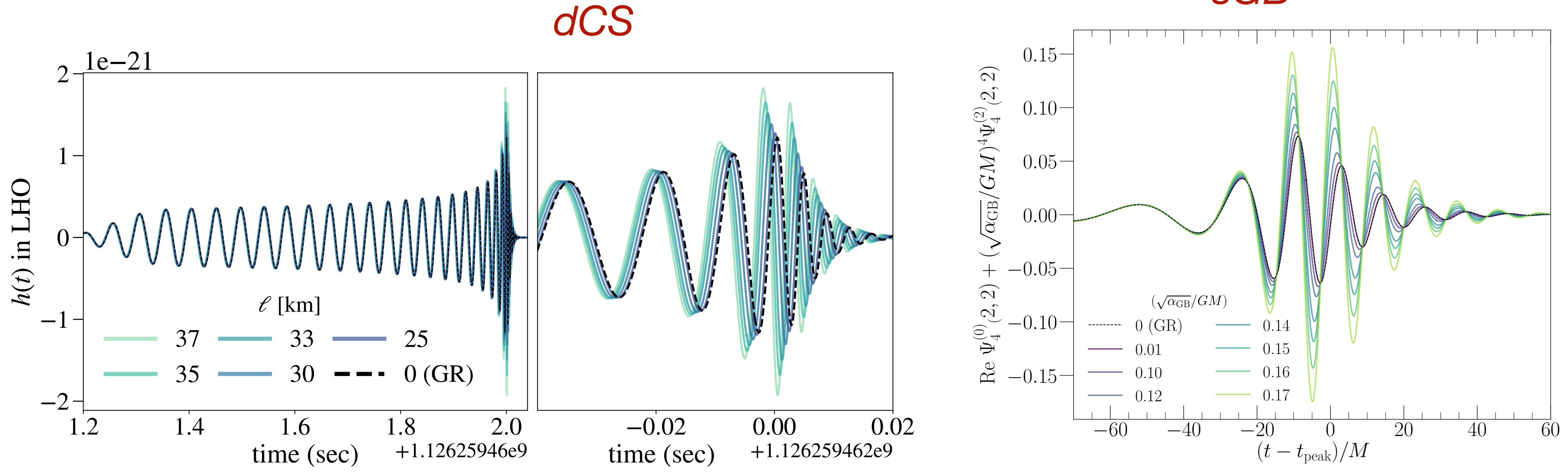
  Problem: secular growth in the waveform!

  (signal of an inconsistency of the order-by-order approach)



Okounkova, PRD '20

dCS *sGB*

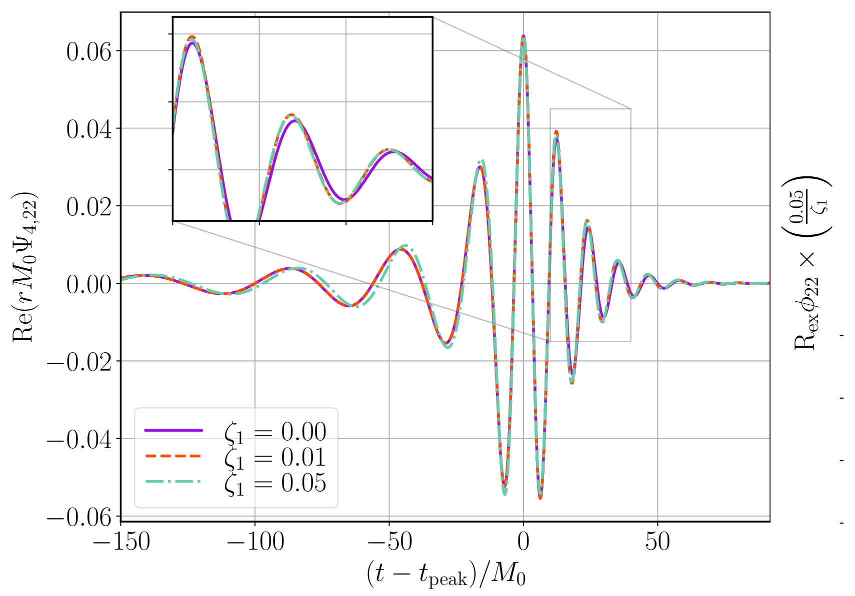


Okounkova, PRD 2022

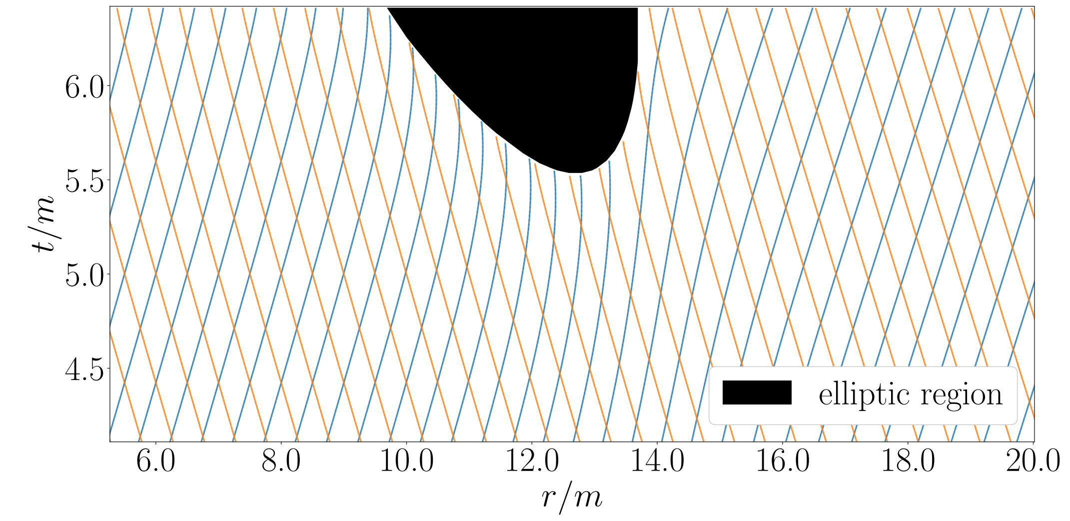
- Small (but finite) coupling, sGB gravity

  (Ripley & Pretorius 2020; Corman et al. 2022; Aresté Saló et al. 2022; see also the review Ripley 2022)

  Problem: elliptic regions appear
  unless the coupling is very small!



Corman et al., '22



(a) EdGB characteristics

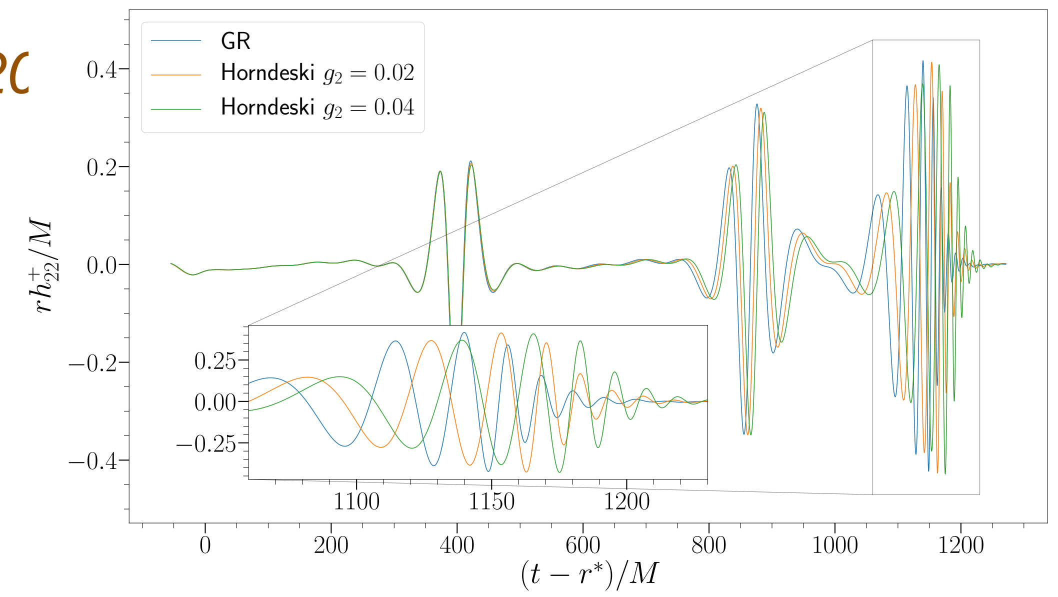
Ripley & Pretorius, 2019

- this list is not exhaustive: there are also:

  - NR simulations of a sGB scalar field on a GR BBH background, useful to learn what's going on, not to provide actual waveforms to compare with observational data *(Silva et al. 2021, Elley et al. 2022, Doneva et al. 20*

  - NR simulations of BBHs in cubic Horndeski gravity: well posed, no-hair theorem apply *(Figueras & França, 2022)*

  No stationary hairy BH exist; scalar field eventually "eaten" by the BH.
  However, if the field is massive, a long-lived scalar cloud may survive long enough to affect the waveform!



BBH merger beyond GR: NR simulations

What did we learn from all this work? Which conclusions can we draw from the problems encountered (secular growth, elliptic regions)?

- NR simulations in sGB gravity are presently possible at least for one class of theories with non-Kerr BHs: sGB gravity

- for these theories, NR evolution seems to break down when the curvature becomes too large

- this breakdown can be interpreted as the theory entering a strong coupling regime, in which higher-order corrections in the EFT expansion can not be neglected
  (Hegade et al. 2023)

- thus, it is not clear if such theories can even lead to detectable modifications to the BBH waveform

Summarizing:

BHs are the best probes for GR deviations in the strong field, large curvature regime.

In order to use the wealth of data we expect from next generation of GW detectors (ground and space)
we need to improve our theoretical modelling of BHs beyond GR.

It would be important to find out the BBH gravitational waveform for some specific theory, but this is a challenging task:
for the modified gravity theories proposed so far:

- for the simplest theories, BHs are the same as in GR
- for the most complex theories, we are very far away from modelling BHs (and even more from modelling BBH coalescence)
- what remains are theories with corrections in the large curvature regime
- for these theories we have some results for inspiral, merger and ringdowm but there are various problems
- the most severe of them is arguably the fact that curvature modifications are highly suppressed with the BH mass
- we still have to study a lot: trying to better understand theories with curvature corrections, and modelling BHs and BBH coalescences for other classes of theories